Lynch Hill School

Primary Academy

How we teach calculations:

Calculation Policy for Mathematics

Revised September 2021

About our Calculation Policy

This calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

Age stage expectations

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014 and should be used alongside the Curriculum's programmes of study. However it is **vital** that all mathematics is driven by the following aims:

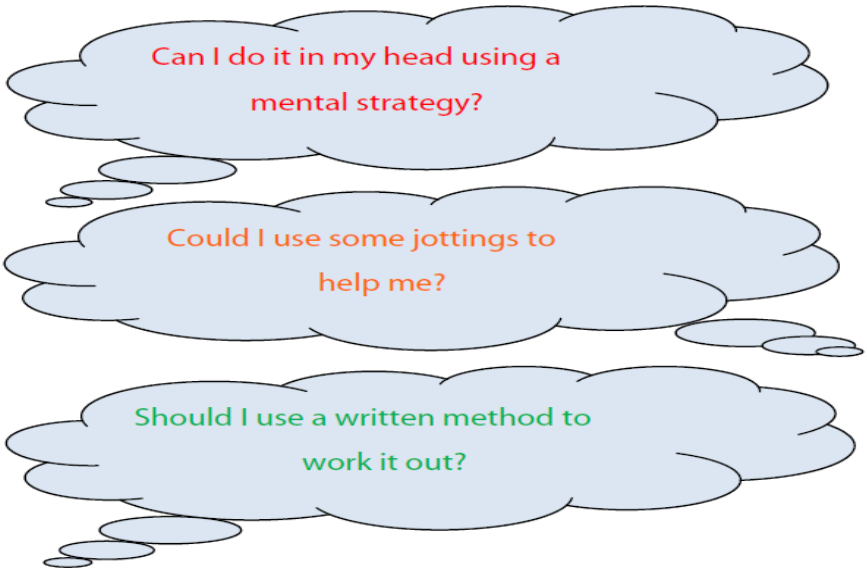
- all pupils become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- all pupils **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- all pupils can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. *(The National Curriculum in England, DfE 2013)*

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons. Concepts should be taught within a context. Problem solving should not be an "add-on".

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:



Year 1 Add with numbers up to 20

Children should know that addition can be **aggregation** (combining 2 sets) and **augmentation** (increasing the size of the set)

Once they can 'count all' they should move to counting on from the first number then to be more efficient, from the largest number)

This can be started with a number line:

Children should then count on without a number line so 6+3 becomes 6, 7,8,**9**

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

  8 + 3 = □ 15 + 4 = □ 5 + 3 + 1 = □ 6 = □+□

It is important that children understand that **=** means 'is the same as'

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting 2 then counting 3.

8+5

Key vocabulary:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Y1:

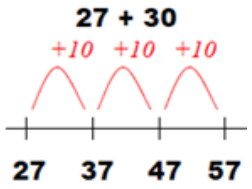
- Read and write numbers to 100 in numerals, incl. 1—20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Video clips: Using a range of equipment and strategies to reinforce addition statements / bonds to 10

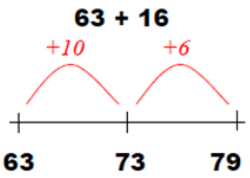
Year 2 Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, leading to more formal methods.

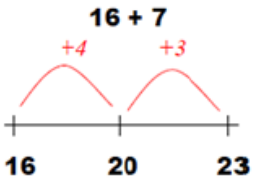
Add 2-digit numbers and tens:



Add pairs of 2-digit numbers,



Add 2-digit numbers and ones:



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

- The next stages is to record mental methods using partitioning into tens and ones separately

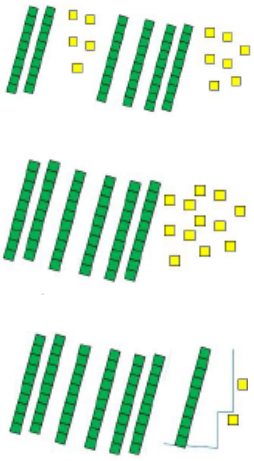
Partitioning into tens and ones.

- Add the tens and then the ones to form partial sums and then add these partial sums
- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.
- This method can be extended for 2-digit + 3-digit and 3-digit + 3-digit and beyond.

Children should use a range of practical apparatus (place value cards, Dienes apparatus, place value counters) to complete 2-digit + 2-digit. They partition the number into tens and ones before adding the numbers together, finding the total.

Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1p, 10p and £1) as place value equipment to help children develop their understanding of a range of representations.

25 + 47



Children may take these jottings to support their calculation.

47 + 76
40 + 70 = 110
7 + 6 = 13
110 + 13 = 123

or

7 + 6 = 13
40 + 70 = 110
110 + 13 = 123

Or

47 + 70 = 117
117 + 6 = 123

Key vocabulary:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary

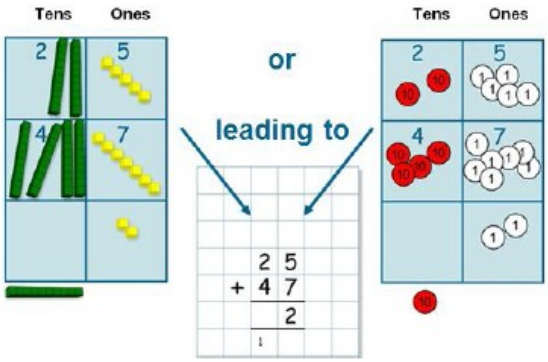
Key skills for addition at Y2:

- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 37)
- Add three single-digit number (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the communicative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

+ Year 3 Add numbers with up to 3-digits

Using Cuisenaire/place value counters alongside columnar written method

- To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.
- Children first experience the practical version of column addition and when confident in explaining this, including exchanging when crossing the tens barrier with ones, they record the written method alongside.
- Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.



Represented in place value columns and rows. Starting adding with the 'least significant digit'

When the tens barriers is crossed in the 'ones' then exchange takes place

Because of the exchange we can now see that this ten belongs in the tens column and is carried there to be included in the total of that column.

The tens are then added together 20 + 40 + 10 = 70, recorded as 7 in the tens column.

Key vocabulary:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact

Key skills for addition at Y3:

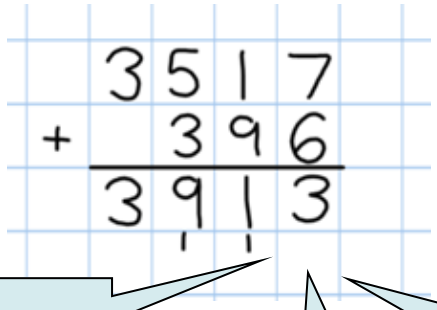
- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- Add a three-digit number and ones mentally (175 + 8)
- Add a three-digit number and tens mentally (249 + 50)
- Add a three-digit number and hundreds mentally (318 + 400)
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit number (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, i.e.. Number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partition and recombining.

See : PowerPoint : Calculation with place value counters (addition) in Staff Common

Year 4 Add numbers with up to 4-digits

Continue with the column method, **adding ones first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913



Add ones first.

'Carry' numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.

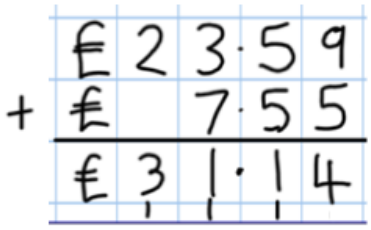
Key vocabulary:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', thousands, hundreds, digits, inverse

Key skills for addition at Y4:

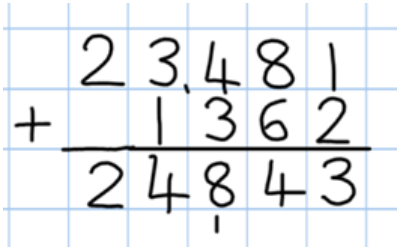
- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the neatest 10, 100 or 1000.
- Estimate and use inverse operation to check answers.
- Solve 2-step problems in context, deciding which operation and method to use and why.
- Find 1000 more or less that a given number.
- Continue to practise a wide range of mental addition strategies, ie. Number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near double, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operation and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

+ Year 5 Add numbers with more than 4-digits

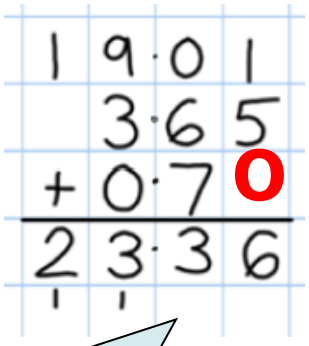
Including money, measures and decimals with different numbers of decimal places.



The decimal point should be aligned in the same way as the place value columns, and must be in the same place in each row.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.

Reinforce correct place value by reminding them the actual value is 1 hundredths add 5 hundredths, **not 1 add 5** for example.

Say '6 tenths add 7 tenths' to reinforce place value.

Children should:

- Understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places.

Key vocabulary:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

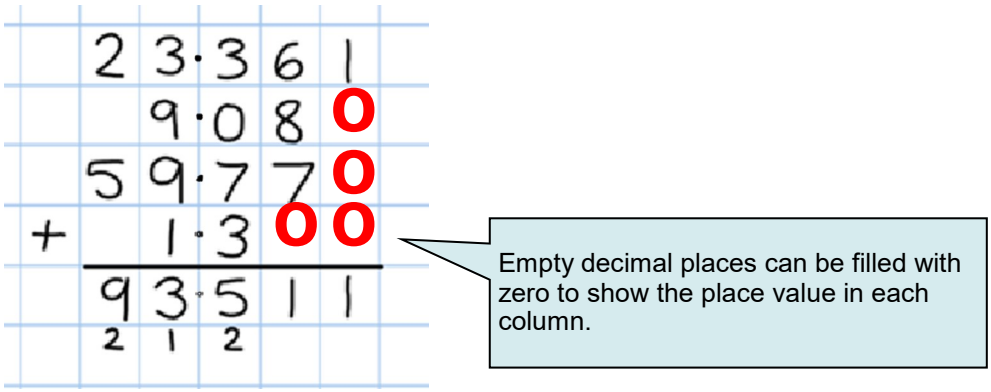
Key skills for addition at Y5:

- Add numbers mentally with increasingly large number, using and practising a range of mental strategies ie. Add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, decided with operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000 and 100,000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

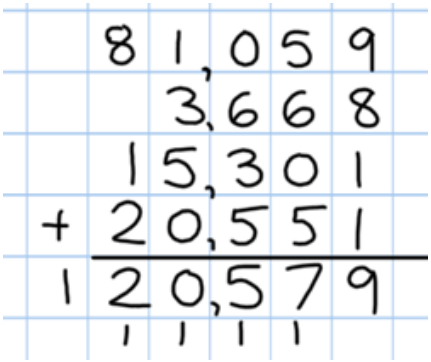
Year 6 Add several number of increasing complexity

Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.



Adding several number with more than 4 digits.



Key vocabulary:
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6:

- Add numbers mentally with increasingly large number, using and practising a range of mental strategies ie. Add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, decided with operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1000000 to the nearest 10, 100, 1000, 10000 and 100000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

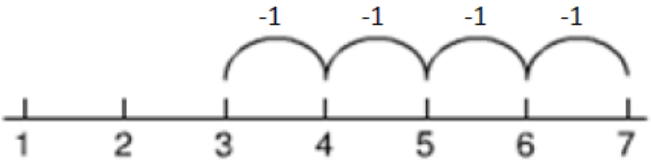
Year 1 Subtract numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with - and = signs.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20:

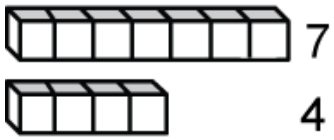


$7 - 4 = 3$

Model subtraction using hundred squares and numbered number lines/ tracks practically.

Find the 'difference between'

This will be introduced practically with the language 'find the difference between' and 'how many more?' in a range of familiar contexts.



'Seven is 3 more than four'

'I am 2 years older than my sister'

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Key vocabulary:
equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_ ?

Key skills for subtraction at Y1:

- Given a number, say **one more or one less.**
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie. Bead string, objects, cubes) and pictures, and missing number problems.
- Read and write number from 0 to 20 in numerals and words.

Year 2 Subtract with 2-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

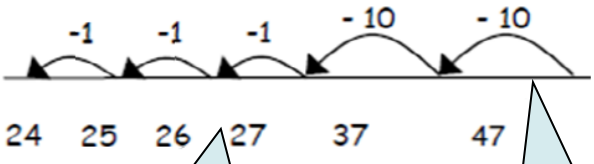
This strategy will be used for:

- 2-digit numbers subtract ones (by taking away / counting back) e.g. 36-7
- 2-digit numbers subtract tens (by taking away / counting back) e.g. 48-30
- Subtracting pairs of 2-digit number (see below:)

Use Dienes blocks for subtraction calculations too.

Subtracting pairs of 2-digit numbers on a number line:

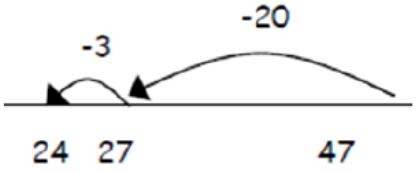
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



Then subtract ones.

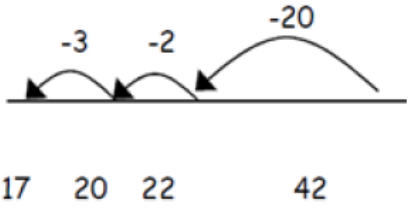
Subtract tens first.

Move towards more efficient jumps block, as below:



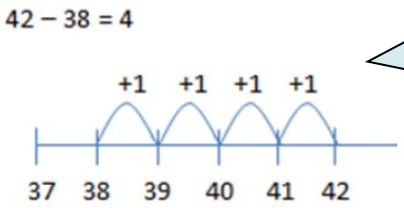
Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example 42-25:



Mental strategy - subtract numbers close together by **counting on:**

Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction.



Start with the smaller number and count on to the largest.

Key vocabulary:
equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units

Key skills for subtraction at Y2:

- Given a number, say **one more or one less.**
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie. Bead string, objects, cubes) and pictures, and missing number problems.
- Read and write number from 0 to 20 in numerals and words.

Year 3 Subtract with 2 and 3-digit numbers

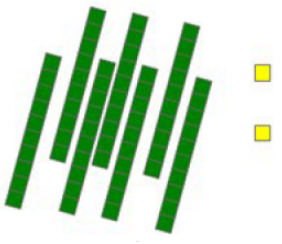
Practical equipment using exchange to 'take away'. Previous experience of **exchange games** is helpful, first with Dienes then place

- Children use practical apparatus to take away the smaller number from the larger number. This should be used to model exchanging as in the example.
- Children's place value knowledge should be good enough to understand that the change still represents the original starting number and is just a different way of partitioning it.

When learning to 'exchange', explore 'partitioning in different ways' so that pupils understand that when you exchange, the VALUE is the same i.e. 72 = 20 + 2 = 60 + 12 = 50 + 22 etc. Emphasise that the value hasn't changed, we have just partitioned it in a different way.

STEP 2: Introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

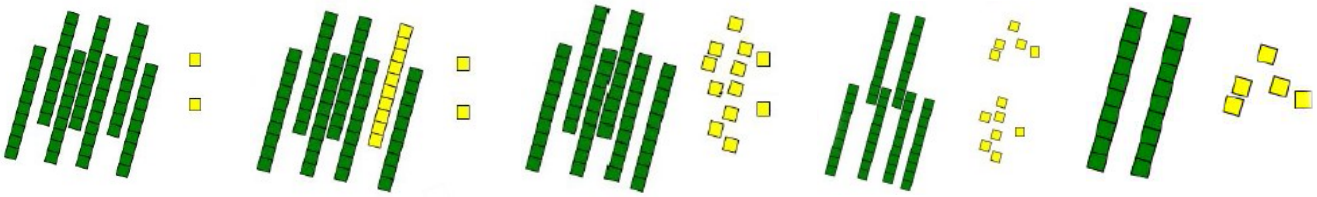
72 - 47



Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten ones. Then subtract 7, and subtract 4 tens.

72 - 47 = 25

This is now "Sixty-twelve" $^{6}\cancel{7}\ ^{1}2$



This stage should also be represented using place value counters, going through the same process as the Dienes example.

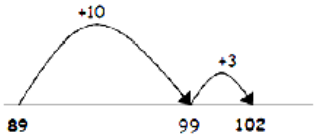
Because of the cumbersome nature of 'exchanges' in this form, examples that children are expected to do with the practical equipment should be limited to 3-digit - 3-digit with one exchange in each calculation

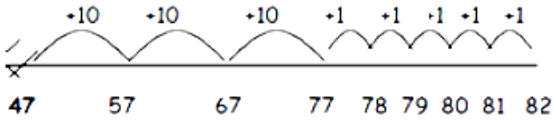
Counting on as mental strategy for subtraction:

Continue to reinforce on as a strategy for close-together numbers (e.g. 121-118) and also for numbers that are 'nearly' multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 1131-79, or calculating change from £1 etc.).

- Start at the smaller number and count on in tens first, then count on in units to find the rest of the difference:

Because counting on in tens is the way we use a 100 square





Key vocabulary:
equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit

Key skills for subtraction at Y3:

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 and 100 more or less than a give number.
- Recognise the place value of each digit in a 3-digit number.
- Counting up difference as a mental strategy when number are close together or near multiples of 10 see examples above)
- Read and write number up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

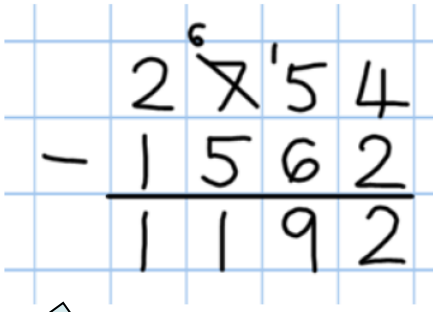
Video clips: 1 Subtraction-teaching children to consider the most appropriate methods before calculating
2 Introduction partitioned column subtraction method, from practical to written

Year 4 Subtract with up to 4-digit numbers column subtraction with 'exchanging' (decomposition):

Making the link between the practical and columnar subtraction

- To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.
- Children first experience the practical version of column subtraction and when confident in explaining this, including exchanging when 'not having enough to subtract from', the record the written method alongside
- Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.

Column subtraction (see video 3)



Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved - mental, counting on, counting back or written method (see video)

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where number are closer together, or where it is easier to count on (see video 1 below).

Key vocabulary:
equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse

Key skills for subtraction at Y4:

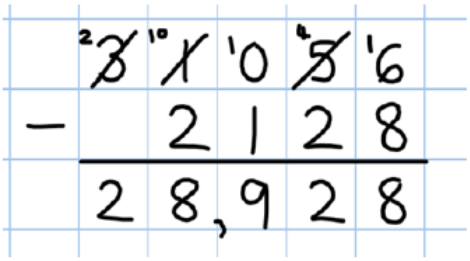
- Subtract by counting on where number are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less that a give number.
- Count backwards through zero, including negative number.
- Recognise place value of each digit in a 4-digit number
- Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Video clips: 1 Subtraction-teaching children to consider the most appropriate methods before calculation
2 Introduction portioned column subtraction method, from practical to written
3 Moving to the compact column method of subtraction (youtube)

Year 5 Subtract with at least 4-digit numbers including money, measures, decimals

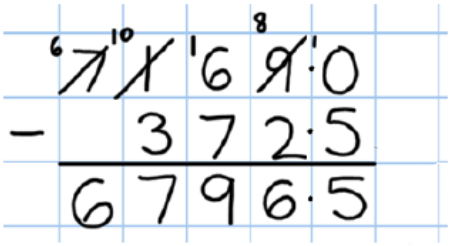
Column subtraction
(with 'exchanging').

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.



Subtracting with larger integers.

See 'moving to the compact method' video.



Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Create lots of opportunities for subtracting and finding differences with money and measures.

Add a 'zero' in any empty decimal places to aid understanding of what to subtract in that column.

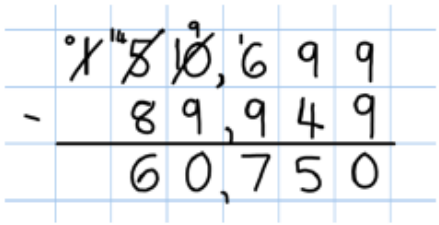
Key vocabulary:
equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Y5:

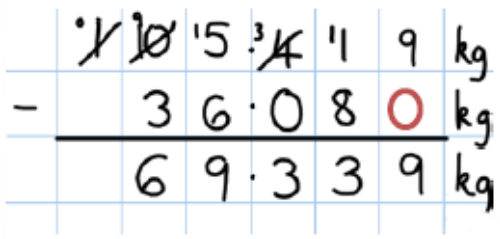
- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers though 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10,000 and 100,000.

Video clip: Moving to the compact column method of subtraction (youtube)

Year 6 Subtracting with increasingly large and more complex numbers and decimal values.



Using the column method to subtract more complex integers



Using the column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skill, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Key vocabulary:
equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

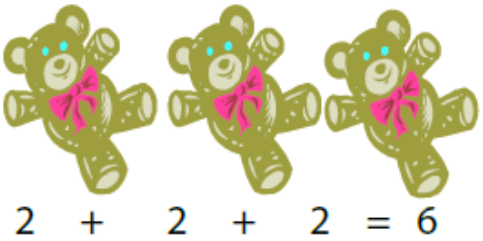
Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Use negative number in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

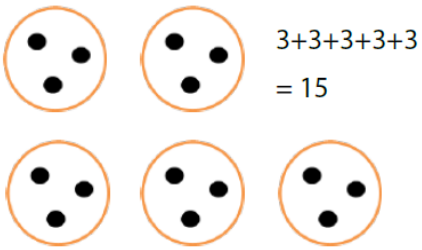
See previous videos for introducing the compact column method.

Year 1 Multiply with concrete objects, arrays and pictorial representations.

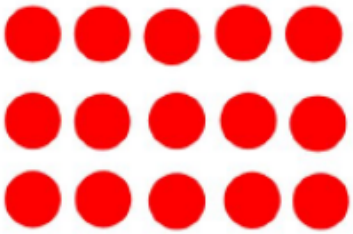
How many legs will 3 teddies have?

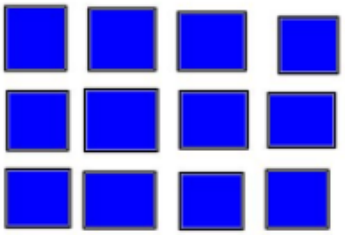


There are 3 sweets in one bag. How many sweets are in 5 bags altogether?



- Give children experience of counting equal group of objects in 2s, 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.





NB. There is no need to introduce the X symbol yet.

Key vocabulary:
Groups of, lots of, times, array, altogether, multiply, count

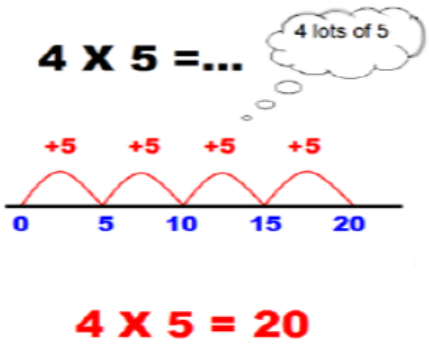
Key skills for multiplication at Y1:

- Count in multiples of 2,5 and 10.
- Solve 1-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connection between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

Year 2 Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Use repeated addition on a number line:

- Starting from zero, make equal jumps on a number line to work out multiplication facts and write multiplication statements using X and = signs.



Use arrays:

3 x 5 = 15

5 x 3 = 15

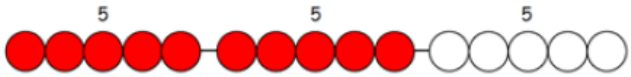
5 x 3 = 3 + 3 + 3 + 3 = 15

3 x 5 = 5 + 5 + 5 = 15

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as 3 x _ = 6.

5 x 3 = 5 + 5 + 5

User practical apparatus (bead string):



Use mental recall:

- Children should begin to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding the operation.

Key vocabulary:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Key skills for multiplication at Y2:

- Count in steps 2,3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens.
- Write and calculate statements using the x and - signs.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication fact.
- Pupils use a variety of language to discuss and describe multiplication.

Video clips: Teaching for understanding of multiplication facts (youtube)
Practical multiplication and the commutative law (youtube)

Year 3 Multiply 2-digits by a single digit number

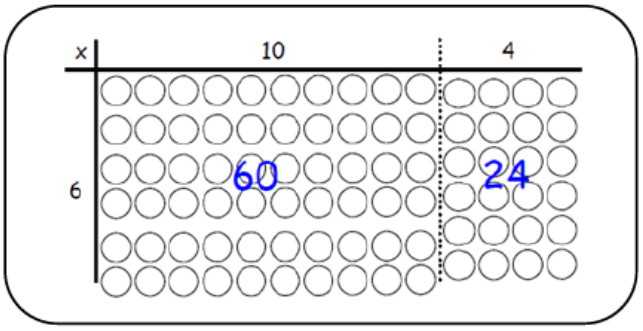
Introduce the grid method for multiplying 2-digit by single-digits:

E.g. 23 x 8 = 184

X	20	3
8	160	24

160 + 24 = 184

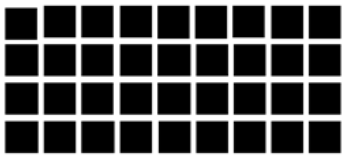
Link the layout of the grid to an array initially:



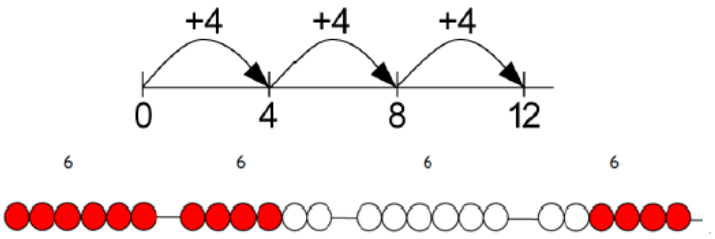
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

To do this, children must be able to:

- Partition numbers into tens and ones
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value.
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



9 x 4 = 36



Key vocabulary:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, ones, value

Key skills for multiplication at Y3:

- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables, and multiply multiples of 10.
- Write and calculate statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Solve simple problems in contexts, deciding which operation and methods to use.
- Develop efficient mental methods to solve a range of problems e.g. using commutativity (**4 x 12 x 5** = 4 x 5 x 12 = **20 x 12** = 240) and for missing number problems □ x 5 = 20, 3 x □ = 18, □ x □ = 32

Video clips: Teaching the grid method as an interim step (partitioning and counters to introduce grid)

Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12

Developing the grid method:

Eg. 136 x 5 = 680

X	100	30	6
5	500	150	30

500
150
+ 30
680

500
150
+ 30
680

Encourage column addition to add accurately.

Move onto short multiplication (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in 'carrying' for written addition.

Children should be able to:

- **Approximate before they calculate,** and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. E.g.

  "346 x 9 is approximately 350 x 10 = 3500"

  Record an approximation to check the final answer against.

- Multiply multiplies of ten and one hundred by a single-digit, using their multiplication table knowledge.

- Recall all times table up to 12 x 12

Key vocabulary:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, sets of, inverse

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for all **multiplication table up to 12 x 12.**
- Recognise place values of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, ore to multiply 3 number.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3, 2 x 6 x 5 = 10 x 6, 39 x 7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000.
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

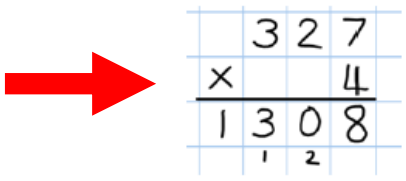
Year 5 Multiply up to 4-digits by **1 or 2** digits.

Introducing column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are fewer steps involved in the column method (see video)
- Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding: 72 x 38** is approximately 70 x 40 = **2800**, and use the approximation to check the reasonableness of their answer against.

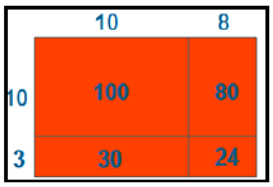
Short multiplication for multiplying by a single digit

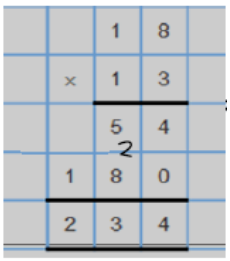
x	300	20	7
4	1200	80	28



Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and difference? Unpick the steps and show how it reduces the steps.

Introduce **long multiplication for multiplying by 2 digits**



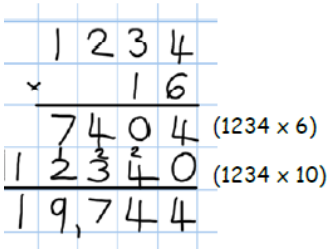


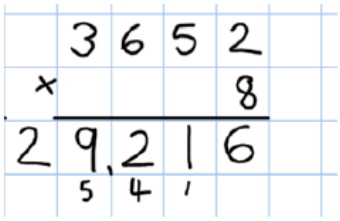
18 x 3 on the 1st row
(8 x 3 = 24, carrying the 2 for twenty, then '1' x 3).

18 x 10 on the 2nd row, Put a zero in units first, then say 8 x 1, and 1 x 1.

The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.

Moving towards more complex numbers:





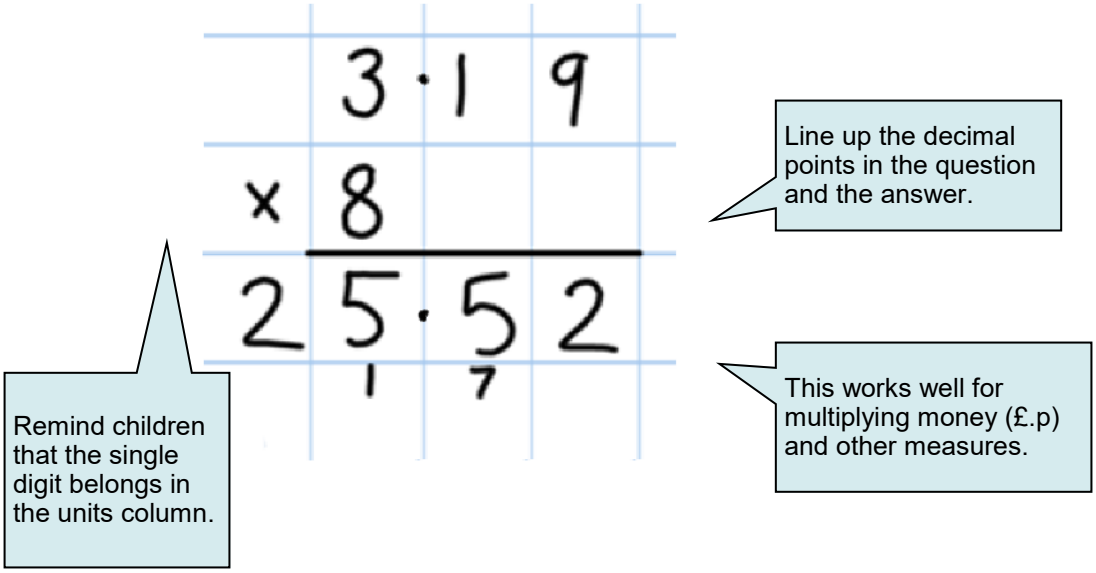
Key vocabulary:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of multiplication tables to 12 x 12.
- Solves problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube number and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Video clips: Moving from grid method to a compact method
Reinforcing rapid times table recall
Demonstration of long multiplication

Year 6 Short and long multiplication as in Y5, and multiply decimals with up to 2d.p by a single digit.



Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit;** to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit.**
- Use **multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number.**

Key vocabulary:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', tenths, hundredths, decimal

Key skills for multiplication at Y6:

- Identify multiples and factors, using knowledge of multiplication tables to 12 x 12.
- Solves problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube number and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Video clips: Moving from grid method to a compact method (youtube)
Reinforcing rapid times table recall (youtube)
Demonstration of long multiplication (SLEP)

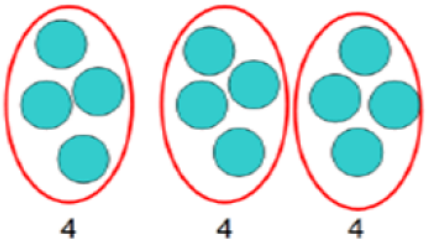
÷ Year 1 Group **and** share small quantities

Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

How many groups of 4 can be made with 12 stars? = 3

Grouping:

Sharing:



12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement…?

"18 shared between 6 people gives you 3 each."

Pupils should:

- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the different between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find half a group of objects by sharing into 2 equal groups.

NB. There is no need to introduce the ÷ symbol yet.

Key vocabulary:
Share, share equally, one each, two each…, group, groups of, lots of, array

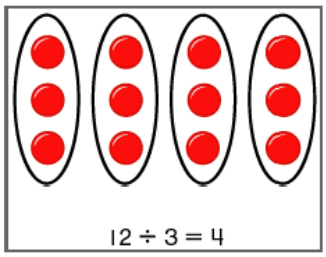
Key number skills needed for division at Y1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representation arrays with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand, division and finding simple fractions of objects, numbers and quantities.
- They make connection between arrays, number patterns, and counting in twos, fives and tens.

÷ Year 2 Group **and** share, using the ÷ and the = sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

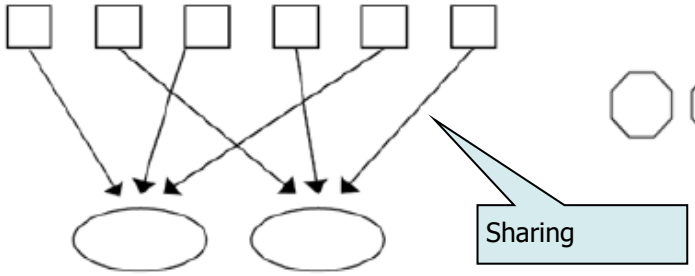


This represents 12 ÷ 3, posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally.

Know and understand sharing **and** grouping:

6 sweets shared between 2 people, how many do they each get?



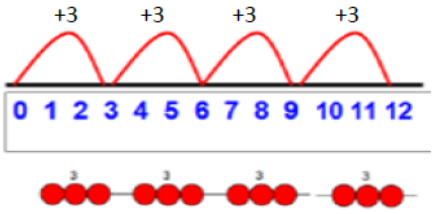
Grouping

There are 6 sweets, how many people can have 2 sweets each?

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out 'how many groups of _ in _ ?'. Pupils could and using a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?' **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

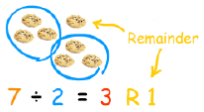
Pose **12 ÷ 3** as **'How many groups of 3 are in 12?'**

Key vocabulary:
Share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Key number skills needed for division at Y2:

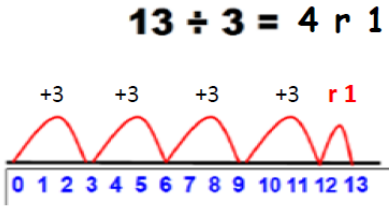
- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

÷ Year 3 Divide 2-digit numbers by a single digit (where there is no remainder in the final answer)



Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

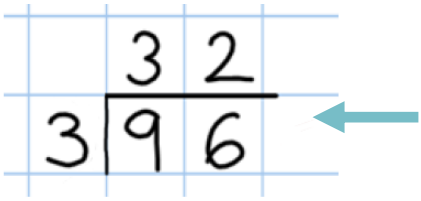
Grouping on a number line:



STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero.

They are also now taught the concept of **remainders**, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

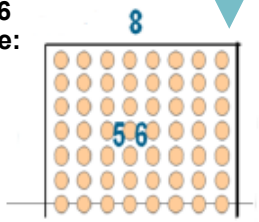
Short division:
Limit numbers to **NO** remainders in the answer **OR** carried (carried digit must be a multiple of the divisor).



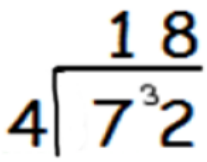
STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introduction the layout of short division by comparing it to an array.

Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

- How many 3s in9? = 3, and record it above the 9 tens.
- Haw many 3s in 6? = 2, and record it above the 6 units.



Short division:
Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the calculation.



STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96 ÷ 4), and be taught to 'carry' the remainder onto the next digit. If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.

Step 3 only taught when pupils can calculate 'remainders'

Key vocabulary:
Share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

Key number skills needed for division at Y3:

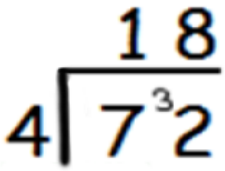
- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for 2-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 ×2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 x 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculation of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Year 4 Divide up to 3-digit numbers by a single digit (without remainders initially)

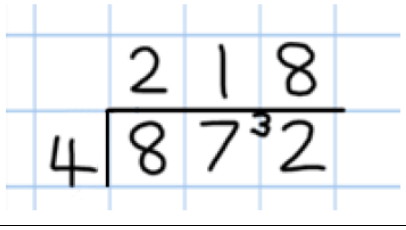
Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Short division should only be taught once children have secured the skill of calculation 'remainders'.

Continue to develop short division:

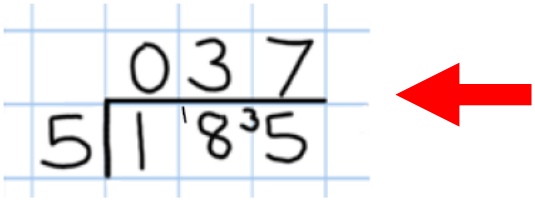


STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (those that do not result in a final remainder - see steps in Y3), but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process (see example).



STEP 2: Pupils move onto dividing number with up to 3-digits by a single digit, however problems and calculation provided should not result in a final answer with remainder at this stage. Children who exceed this expectation may progress to Y5 level.

Include money and measure contexts when confident.



When the answer for the first column is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

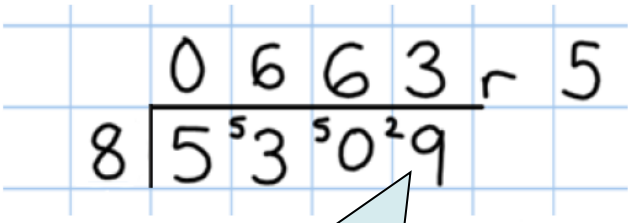
Key vocabulary:
Share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor

Key number skills needed for division at Y4:

- Recall multiplication and division derived facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 x 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder number. This should include correspondence questions such as three cakes shared equally between 10 children.

÷ Year 5 Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:



Short division with remainders: Now that pupils are introduced to examples that give rise to the remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and <u>how</u> to express it**, i.e.. As a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 633 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Include money and measure contexts.

If children are confident and accurate:

- Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation

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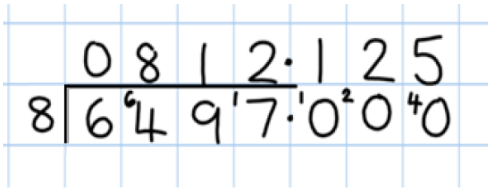
Key vocabulary:
Share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime)

Key number skills needed for division at Y5:

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiple and divide number mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decamped into their factor.
- Multiply and divide whole number and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 24½ ≈ 24.5).
- Solve problems involving combinations of all four operations, including understanding of the equal signs, and including division for scaling by different fractions and problems involving simple rates.

÷ Year 6 Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

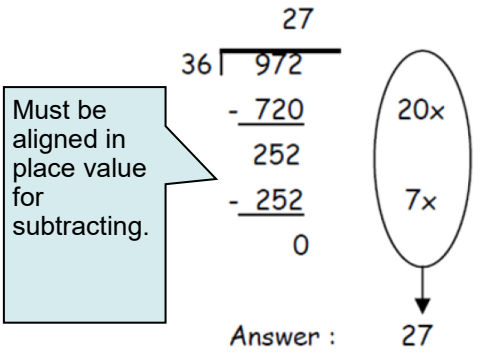
Short division, for diving by a single digit: e.g. 6497 ÷ 8



Short division with remainders: Pupils should continue to use this method, but with number to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculation a decimal remainder: in this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is stall a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce long division by chunking for dividing by 2 digits.



- Find out 'How many 36s are in 972?' by subtracting 'chunks' of 36, until zero is reached 9or until there is a remainder).
- Teach pupils to write a 'useful list' first at the side that will help them decide what chunks to use, e.g.:
  'Useful' list: 1x=36
  10x=360
  100x=3600
- Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots? Can use 100 lots? As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their 'useful' lists.

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Key vocabulary: Share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime), common factor

Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. User short division where appropriate.
- Perform mental calculations, including with mixed operations and larger numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in case where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.